

Chapter 2 — Fractions as Relationships — Teacher Guide

Chapter at a glance

Lesson duration	4 lessons, 40–60 minutes each (see pacing table below)
Materials	Paper strips, fraction circles or rectangles, counters or linking cubes, a 0–2 number line, colored pencils, the four chapter diagrams
Launch question	“Two classes each have three-fourths of their students at an activity. Do they have the same number of students?”
Key representation	The fraction strip paired with the number line — keep one visible throughout instruction
Anticipated responses	Correct: names the whole in each case and computes both amounts. Partial: recognizes the fractions match but assumes the amounts must also match. Incorrect: “ is , so the amounts are the same.”
If the readiness check fails	Reteach equal partitioning and division facts before starting the Launch; do not skip the concrete strip-folding step
Talk prompt	“What is the whole, and what has been divided into equal parts?”
Exit ticket	4 items — identify whole/parts, find a fraction of a quantity, place a fraction greater than 1, explain the operator (see Section 16)
Reteach / extend	Reteach: physical strips before symbols, whole-number wholes only. Extend: construct two contexts for the same fraction, derive mixed numbers from improper fractions.

Purpose and pacing

Chapter 2 establishes the meanings of fractions that later chapters will use for fraction operations, decimals, percents, ratios, and proportional scaling. Do not teach the numerator and denominator as labels only. Students should repeatedly connect the symbols to a whole, equal parts, a quotient, a location, and an operator action.

Lesson	Focus	Suggested time
Lesson 1	Ready-to-Learn Check,	45–55 minutes

Lesson	Focus	Suggested time
	same fraction/different wholes, equal parts, and fraction bar	
Lesson 2	Number line, quotient meaning, fractions greater than 1, and operator model	45–55 minutes
Lesson 3	Worked examples, misconception analysis, and Worksheet A	40–50 minutes
Lesson 4	Worksheets B and C, Tripwire Check, Exit Ticket, and reteaching/extension	45–60 minutes

Materials

Use paper strips, fraction circles or rectangles, counters or linking cubes, a 0–2 number line, colored pencils, and the four chapter diagrams. If physical manipulatives are unavailable, use folded paper strips and drawn groups. All essential meaning should remain available in black-and-white printing.

The chapter routine

Use the same routine introduced in Chapter 1:

Understand → Represent → Predict → Choose a method → Solve → Check → Explain.

For Chapter 2, “represent” should usually mean a fraction strip, area model, number line, equal groups, or operator diagram. Students should not be required to use all representations every time; they should learn to choose a representation that clarifies the meaning.

Launch guidance

Show $\frac{1}{2}$ of 20 and $\frac{1}{3}$ of 12 using two different-sized sets or strips. Ask students to decide whether the amounts are equal before calculating. Listen for the misconception that the same fraction must produce the same amount. Use the launch to establish the distinction between **same relationship** and **same quantity**.

Do not begin with the words numerator and denominator. Let students describe “four equal parts” and “three selected parts” first. Then name the conventional terms.

Teacher prompts

Purpose	Prompt
Identify the whole	“What complete quantity is being divided or measured?”
Identify equal parts	“Would each part have the same size or value?”

Purpose	Prompt
Connect notation to meaning	"What does the fraction bar mean here?"
Support number-line reasoning	"How many equal intervals make one whole?"
Support operator reasoning	"How many equal groups does the denominator make? How many groups does the numerator select?"
Check magnitude	"Is the fraction less than 1, equal to 1, or greater than 1? What should that tell you about the answer?"
Compare contexts	"Is the relationship the same? Is the whole the same?"
Encourage explanation	"What would you say to a student who used only the numerator?"

Anticipated errors and responses

Student response	Likely meaning	Repair move
" means 3."	The numerator is being treated as the amount.	Build 20 into four equal groups and ask for the size of one fourth.
"The denominator is the answer."	The number of groups is being confused with the selected quantity.	Ask the student to label the size of one group and the number of groups selected.
"Three pieces always means ."	Equal partitioning is not understood.	Compare equal and unequal strips; ask whether the pieces represent equal shares.
" is impossible."	Fractions are being restricted to numbers less than 1.	Place and on a number line and rename as 1 .
" of 20 equals ."	The operator action is not understood.	Read the problem aloud as "three-fourths of twenty" and model the four groups.
" means $4 \div 3$."	Fraction order is reversed.	Ask what is being shared and among how many groups; preserve units.
"The same fraction gives the same amount."	Relationship and quantity are being conflated.	Compare of 12 and of 20 with two models.

Differentiation

Students needing concrete support

Use physical strips or counters before symbols. Keep the whole visible and label it explicitly. Ask students to build the denominator first, then shade or select the numerator. For fraction-of-a-number problems, make equal groups with counters and say the unit fraction aloud: “One fifth of 30 is 6.” Then select the numerator number of groups.

Students needing language support

Use sentence frames:

- “The whole is _____.”
- “The denominator tells me _____.”
- “The numerator tells me _____.”
- “_____ means _____ equal parts out of _____.”
- “_____ is greater than 1 because _____.”
- “The answer is reasonable because the fraction is _____ than 1.”

Students ready to extend

Ask students to construct two contexts for the same fraction, compare equivalent fractions on a number line, derive a mixed number from an improper fraction, and explain why $\frac{a}{b}$ of n can be represented as $(a \times n) \div b$ when the quantities are compatible.

Mastery criteria

A student has mastered Chapter 2 when the student can identify the whole and explain the roles of the numerator and denominator. The student can represent a fraction in at least three connected ways and place it accurately on a number line. The student can find a fraction of a quantity and diagnose the two central misconceptions: ignoring the whole, and accepting unequal parts.

A numerical answer without a meaning, model, or reasonable check is not sufficient evidence of full mastery when the task is conceptual.

Answer-key policy for this chapter

For open responses, accept equivalent language when the mathematical meaning is correct. A complete answer should identify the whole, preserve equal partitioning, use units where appropriate, and explain the operator action when finding a fraction of a quantity.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection:

1. Place $\frac{3}{5}$ on a number line.
2. Name the whole in the statement “3 out of 5 equal parts.”
3. Compare $\frac{3}{5}$ and $\frac{2}{3}$, using a common benchmark or a diagram.
1. Explain why the same fraction can name different amounts when the wholes differ.

Strategy Studio guidance:

1. Expected ideas: ;
2. The whole must be named;
3. because ;
4. The denominator describes equal partitioning but does not alone determine size.
Ask students to redraw both fractions with the same whole.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: .

Altered: and , so is smaller.

Non-routine: the fraction names a relationship, so the whole must be identified. Repair: denominator size alone does not determine fraction size; equal partitioning and the numerator both matter.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Launch: The same fraction, different wholes

1. Yes, both are three-fourths.
2. No. Class A has 15 participating students; Class B has 9.
3. Class A has more because three-fourths of 20 is greater than three-fourths of 12.
4. The whole is the complete class: 20 students in Class A and 12 students in Class B.
5. The denominator says the whole is divided into 4 equal parts. The numerator says 3 of those parts are selected.

Answer Key

Ready-to-Learn Check

1. 8.
2. 6.
3. A correct drawing has four equal areas.
4. The whole is the complete amount or quantity being divided or measured.
5. Halfway between 0 and 1.
6. No. Equal parts must have the same size or value.

Concrete model: build equal parts

1. .
2. .
3. The pieces are not equal-sized, so the three shaded pieces do not represent three-fourths of one whole.
4. The denominator tells how many equal groups or parts make the whole; it does not give the final amount by itself.

Number-line practice

The order is 0, , , 1, . The point is to the right of 1 because $\frac{3}{4} = 1$ and one additional fourth gives $\frac{4}{4} = 1$.

Worked examples

Worked Example A: means 3 of 4 equal parts, $3 \div 4$, and the point three-fourths of the way from 0 to 1.

Worked Example B: $40 \div 5 = 8$, then $8 \times 3 = 24$.

Worked Example C: The student is incorrect; $20 \div 4 = 5$ and $5 \times 3 = 15$.

Worked Example D: of 20 = 15; of 40 = 20; therefore of 40 is greater.

Worksheet A

1. **The denominator (5) tells the number of equal parts in one whole.** Likely wrong answer: calls 5 “the answer” without connecting it to equal parts. Misconception: not distinguishing parts-in-the-whole from parts-selected. Next move: “how many equal groups does the whole split into?”
2. **The numerator (3) tells the number of equal parts selected.** Likely wrong answer: describes 3 as a stand-alone amount, not a count of parts. Misconception: numerator treated as independent from the whole. Next move: “3 out of how many?”
3. **$= 2 \div 7$; two wholes shared equally among 7 groups.** Likely wrong answer: reverses the division, writing $7 \div 2$. Misconception: numerator/denominator order confused with dividend/divisor order. Next move: “which number is being split — the 2 or the 7?”
4. **Divide 0 to 1 into 7 equal intervals; locate the 2nd point from 0.** Likely wrong answer: counts intervals starting from 1 instead of 0, landing one interval too far. Misconception: off-by-one counting on the number line. Next move: mark and label all 7 tick points before locating the fraction.
5. **Any valid model with 6 equal parts and 4 selected; equivalent to .** Common flaw: draws 6 parts of unequal size, or shades a count other than 4. Next move: check all 6 parts are the same size before checking the shaded count.
6. **No. of 12 = 9, while of 20 = 15.** Likely wrong answer: “yes, because the fraction is the same.” Misconception: assumes equal fractions always give equal amounts, ignoring the whole. Next move: compute both amounts and compare.
7. **(equivalent to).** Likely wrong answer: — reverses which count is parts-selected vs. parts-in-whole. Misconception: not tracking which number is the numerator. Next move: “how many parts make the whole? how many are shaded?”
8. **The pieces are not equal-sized, so three shaded pieces out of four unequal pieces is not .** Likely wrong answer: accepts the label because 3 of 4 pieces are shaded,

without checking equal size.
 Misconception: counting pieces instead of verifying equal partition.
 Next move: "are all four pieces the same size?"

9. **The whole is the class of 30 students.** Likely wrong answer: says "" or "5" is the whole, confusing the fraction with the quantity it acts on. Misconception: not distinguishing the fraction from its whole. Next move: "what complete group is being described?"
10. **Greater than 1; = 1 .** Likely wrong answer: "less than 1, because it's a fraction." Misconception: assuming all fractions are less than 1. Next move: "how many fifths make one whole? how does that compare to 7?"
11. **5.** Likely wrong answer: 4 (divides 20 by 5 instead of by the denominator 4). Misconception: misreads which number is the denominator. Next move: "how many equal groups does 'fourths' make?"
12. **15.** Likely wrong answer: 5 (finds one-fourth and stops, forgetting to multiply by 3). Misconception: the chapter's most common slip — unit-fraction found but never scaled to the numerator. Next move: "you found one group — how many groups does the numerator ask for?"
13. **14.** Likely wrong answer: 7 (finds $35 \div 5$ and stops, without multiplying by 2). Misconception: same unit-fraction-only error as #12. Next move: "that's one-fifth — what is two-fifths?"
14. **is less than 1, so of a positive whole must be less than the whole.** Common flaw: answers with the specific numbers ($15 < 20$) instead of the general reasoning

based on the fraction's size relative to 1. Next move: ask for the explanation using the fraction alone, without plugging in numbers.

Worksheet B

1. **Part of a whole: 5 of 8 equal parts. Quotient: $5 \div 8$.** Likely wrong answer: gives only one meaning, usually just the quotient. Misconception: treating the meanings as interchangeable rather than two connected descriptions. Next move: ask for one "parts" sentence and one "division" sentence.
2. **$< < 1 <$.** Likely wrong answer: places before 1, since its numerator (5) "looks small." Misconception: comparing numerators/denominators visually instead of converting to a common reference. Next move: convert each to a decimal or place each on a 0–2 number line first.
3. **18.** Likely wrong answer: 6 (stops at $42 \div 7$, never multiplies by 3). Misconception: the unit-fraction-only error, the chapter's most common slip. Next move: "that's one-seventh — what is three-sevenths?"
4. **40.** Likely wrong answer: 8 (stops at $48 \div 6$, never multiplies by 5). Misconception: unit-fraction-only error. Next move: "you found one-sixth — the question asks for five-sixths."
5. **63.** Likely wrong answer: 9 (stops at $90 \div 10$, never multiplies by 7). Misconception: unit-fraction-only error, again. Next move: "one-tenth is 9 — what is seven-tenths?"
6. **18 players.** Likely wrong answer: 6 (divides 24 by 4, never multiplies by 3). Misconception: unit-fraction-only error. Next move: "6 is one-fourth of

the team — how many is three-fourths?”

7. **18 books.** Likely wrong answer: 9 (divides 45 by 5, never multiplies by 2). Misconception: unit-fraction-only error. Next move: “9 is one-fifth — what is two-fifths?”
8. **The student used only the numerator (4) instead of computing the operator. Correct answer: 24.** Likely wrong answer: repeats the error already shown in the problem (4), if the student doesn’t recompute independently. Misconception: accepting the numerator as the final answer. Next move: “walk through both steps: divide by the denominator, then multiply by the numerator.”
9. **of 24 = 16; of 20 = 15; of 24 is greater.** Likely wrong answer: compares and directly and concludes of 20 is greater, without computing either amount. Misconception: comparing the fractions instead of the resulting quantities — only valid when the wholes match. Next move: “these act on different wholes — compute both amounts before comparing.”
10. **Answers vary; must state that the same fraction can represent different amounts when the whole changes.** Common flaw: restates “the whole is important” with no explanation or numbers. Next move: ask for one specific example with two different wholes.
11. **Answers vary; the fraction must have numerator greater than denominator and be placed correctly beyond 1.** Common flaw: creates a fraction greater than 1 but places it between 0 and 1 on the number line. Next move: rename it

as a mixed number first, then place it.

12. **Answers vary; the situation must make the whole and the 8 equal parts explicit.** Common flaw: gives a situation without specifying the whole (for example, “of pizza” with no stated number of pizzas or slices). Next move: “of what specific whole, divided into what 8 equal parts?”

Worksheet C

1. **18 students.** Likely wrong answer: 6 (unit-fraction-only error: $30 \div 5$, forgets $\times 3$). Misconception: the chapter’s most common slip — stopping before scaling by the numerator. Next move: “6 is one-fifth of the class — what is three-fifths?”
2. **\$30.** Likely wrong answer: \$10 (finds one-fourth, $40 \div 4$, but doesn’t multiply by 3). Misconception: unit-fraction-only error. Next move: “\$10 is one-fourth of the bill — what is three-fourths?”
3. **2 cups.** Likely wrong answer: 6 cups (multiplies 2×3 and ignores the fraction, or loses track of what the fraction acts on). Misconception: treating “3 batches” as a whole-number multiplier disconnected from the fraction. Next move: “one batch needs cup — what do 3 batches need?”
4. **metre.** Likely wrong answer: metre (adds instead of subtracting) or metre (subtracts numerators and denominators separately: over , skipping a common denominator entirely). Misconception: treating “remains” as addition, or subtracting fractions without a common denominator. Next move: “does ‘how much remains’ mean

- more or less than ? Find a common denominator first.”
5. **They are equal; both equal 18.** Likely wrong answer: assumes they're unequal because the fractions and wholes both differ, without computing. Misconception: assuming different-looking expressions must give different results. Next move: “compute both amounts before deciding.”
 6. **The student is wrong; of 20 = 25, which is greater than 20.** Likely wrong answer: agrees with the student, reasoning “it’s a fraction, so it must make things smaller.” Misconception: assuming every fraction acts like a fraction less than 1. Next move: “is less than or greater than 1? What should that tell you?”
 7. **12 in band; 16 not in band.** Likely wrong answer: finds 12 but forgets the “not in band” part, or stops at $28 \div 7 = 4$ without the final multiplication. Misconception: incomplete multi-step answer combined with the unit-fraction-only error. Next move: “you found one-seventh — how many are three-sevenths? How many are left over?”
 8. **Answers vary; acceptable meanings include 5 of 8 equal parts, $5 \div 8$, or a point at 0.625.** Common flaw: gives two meanings that are actually the same idea reworded (for example, “5 out of 8” and “5 parts of 8”), rather than two genuinely different meanings. Next move: ask for one meaning from the “parts” family and one from the “division/location” family.
 9. **Answers vary; the calculation must actually produce 18.** Common flaw: writes a fraction-of-a-number expression that doesn’t evaluate to 18 — an arithmetic slip. Next move: have the student verify their own answer by recomputing it.
 10. **9. The is the operator acting on 12, not the answer itself.** Likely wrong answer: agrees that $of\ 12 =$, treating the fraction as self-evidently the answer. Misconception: not recognizing “of” as an operation to perform. Next move: “read it as ‘three-fourths of twelve’ — what operation does ‘of’ represent here?”
 11. **20 blue; 4 not blue.** Likely wrong answer: stops at $24 \div 6 = 4$ and reports 4 as the blue count, or finds 20 but forgets “not blue.” Misconception: confusing the unit-fraction value with the final answer, and skipping the last step. Next move: “4 is one-sixth — the question asks for five-sixths blue, and how many are left over.”
 12. **12; representations must include a context, a diagram showing two of three equal groups, the calculation $18 \div 3 \times 2$, and an explanatory sentence.** Common flaw: gives the numeric answer and one representation but skips another, most often the explanatory sentence. Next move: check all four required pieces are present, not just the correct number.
 13. **6 kg. The book’s page count (200) and the preparation time (45 minutes) are not needed to find the amount of sugar.** Likely wrong answer: works one of the irrelevant numbers into the calculation, most often the 45 minutes. Misconception: assuming every number in a word problem must be used. Next move: “what does the question actually ask for? Which numbers answer that?”

14. **Answers vary; must describe a situation where 24 is a whole quantity and selects five of six equal parts of it, with the meaning of both numbers stated.**

*Common flaw: gives a situation but never states what the 24 or the mean specifically. Next move: ask the student to finish the sentences "24 is " **and** " means out of ___ equal parts."*

Extend your thinking

1. Yes. For example, $\frac{1}{2}$; they locate the same point and describe the same value.
2. A fraction is greater than 1 when its point lies to the right of 1; equivalently, its numerator is greater than its denominator.
3. If $\frac{1}{4}$ of a whole is 15, one fourth is 5 and the whole is 20.
4. The fraction is the same relationship, but the wholes differ; of 18 $\frac{1}{2}$ = 9 and of 30 $\frac{1}{2}$ = 15.
5. Answers vary; each context must identify the whole and show four of five equal parts.

Tripwire Check

Part 1: The student is incorrect. $30 \div 5 = 6$, then $6 \times 2 = 12$.

Part 2: The label is invalid because the four parts are not equal-sized. The central

repair is to return to equal partitioning and the operator model.

Exit Ticket

1. The whole is 35; the denominator makes 7 equal groups; the numerator selects 4 groups.
2. $35 \div 7 = 5$; $5 \times 4 = 20$.
3. Greater than 1; $\frac{1}{2} = 1$, and its point lies to the right of 1.
4. $\frac{1}{2}$ is the operator and 15 is the whole being acted upon; of 15 $\frac{1}{2}$ = 7.5, not 10.

Diagnostic notes

- If a student answers with the numerator only, the student needs an operator model with a clearly labeled whole.
- If a student confuses denominator and answer, return to "one denominator-sized group" and ask for the size of one part.
- If a student accepts unequal parts, use two strips with the same number of pieces and compare the sizes of the pieces.
- If a student rejects fractions greater than 1, place $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ on a number line and rename them as mixed numbers.
- If a student can calculate but cannot explain, require a model and sentence before further symbolic practice.

